Settling the Communication Complexity of Combinatorial Auctions with Two Subadditive Buyers

Presented by

Tomer Ezra (1), Michal Feldman (1,2), Eric Neyman (3), Inbal Talgam-Cohen (4), Matt Weinberg (3)

(1) Tel-Aviv University, (2) Microsoft Research, (3) Princeton University, (4) Technion Institute of Technology

tomer.ezra@gmail.com

Motivation

Objective: Maximize Social Welfare in Combinatorial Markets

Markets with 2 sub-additive agents
Communication complexity model

Research agenda

what approximation to optimal social welfare can be achieved with polynomial communication

Is there a gap between truthful and non-truthful mechanisms?

Model

m indivisible items

2 agents (and) with **subadditive** valuation functions $f_A, f_B: 2^{[m]} \to R_+$

α-Approximation:

- Find X (efficiently) such that $f_A(X) + f_B(\bar{X}) \geq \alpha \cdot opt$

½ Approximation is Trivial

Protocol 1: Allocate all items to a Random player

½ Welfare ✓

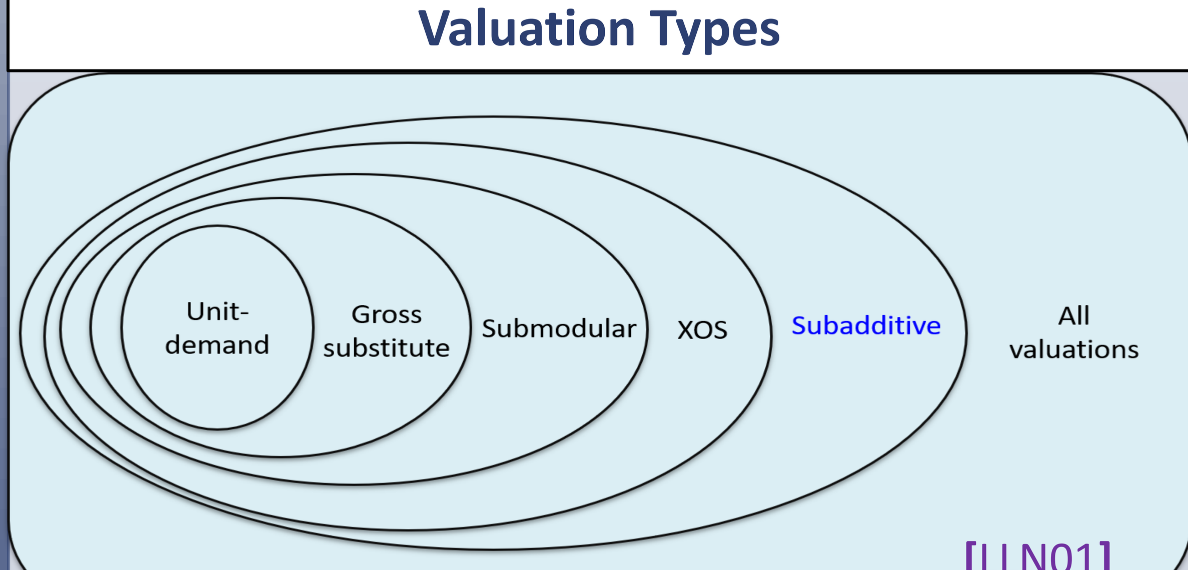
Protocol 2: Alice and Bob send $f_A([m]), f_B([m])$ allocate all items to the player with the higher value

Deterministic ✓

Protocol 3: the same as Protocol 2, but charges the winner a price of $\min(f_A([m]), f_B([m]))$

Truthful ✓

Valuation Types



Submodular	$S \subseteq T \Rightarrow v(x \mid S) \geq v(x \mid T)$
XOS	**Maximum over additive functions**
Subadditive	$v(S) + v(T) \geq v(S \cup T)$

Main Result

Open problem [Fei06]: Can we get better than 1/2 approximation with subexponential communication?

Main result: **No** protocol (deterministic or randomized, truthful or non-truthful) guarantees better than ½ -approximation for **two subadditive** bidders

More precisely:

- $(1/2+\varepsilon)$-approximation requires exp(m) communication
- $(1/2+\omega(1/\log m))$-approximation requires superpoly(m) communication
- $(1/2+O(1/\log m))$-approximation in poly(m) communication
- $(1/2+O(1/\log k))$-approximation in poly(m) communication for MPH-k ∩ subadditive

Of independent interest: new type of subadditive non-XOS valuations

Previous Results

N=2	Submodular	XOS	Subadditive	General
Communication hardness	17/18 [DV13]	3/4 [DNS05]	3/4 [DNS05]	1/2 [NS06]
Communication protocol	13/17 [FV10]	3/4 [Fei06]	1/2	1/2
Truthful comm. protocol	1/2	1/2	1/2	1/2

Corollaries of Main Result

Corollary 1: no separation between truthful and non-truthful mechanisms for 2 subadditive bidders

Corollary 2: price of anarchy of simple mechanisms for two subadditive bidders is 1/2;

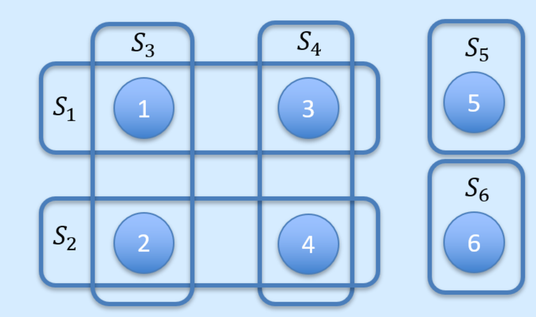
i.e., simultaneous first-price auction is the best simple auction (follows by [Rou14])

Set Cover Function

Let $S = \{S_1, \dots, S_k\}$ be a collection of subsets of $[m]$

Define $f_S(T) = min\{\,|I| : T \subseteq \cup_{i \in I} S_i\}$

i.e. minimum number of sets in S needed to cover T



Modified Set Cover

$f_S(T) < \ell/2$, $f_S(\bar{T}) \geq \ell/2$ ⇓ $f_S^\ell(T) = f_S(T)$, $f_S^\ell(\bar{T}) = \ell - f_S(T)$	$f_S(T) \geq \ell/2$, $f_S(\bar{T}) < \ell/2$ ⇓ $f_S^\ell(T) = \ell - f_S(\bar{T})$, $f_S^\ell(\bar{T}) = f_S(\bar{T})$	$f_S(T) \geq \ell/2$, $f_S(\bar{T}) \geq \ell/2$ ⇓ $f_S^\ell(T) = \ell/2$, $f_S^\ell(\bar{T}) = \ell/2$

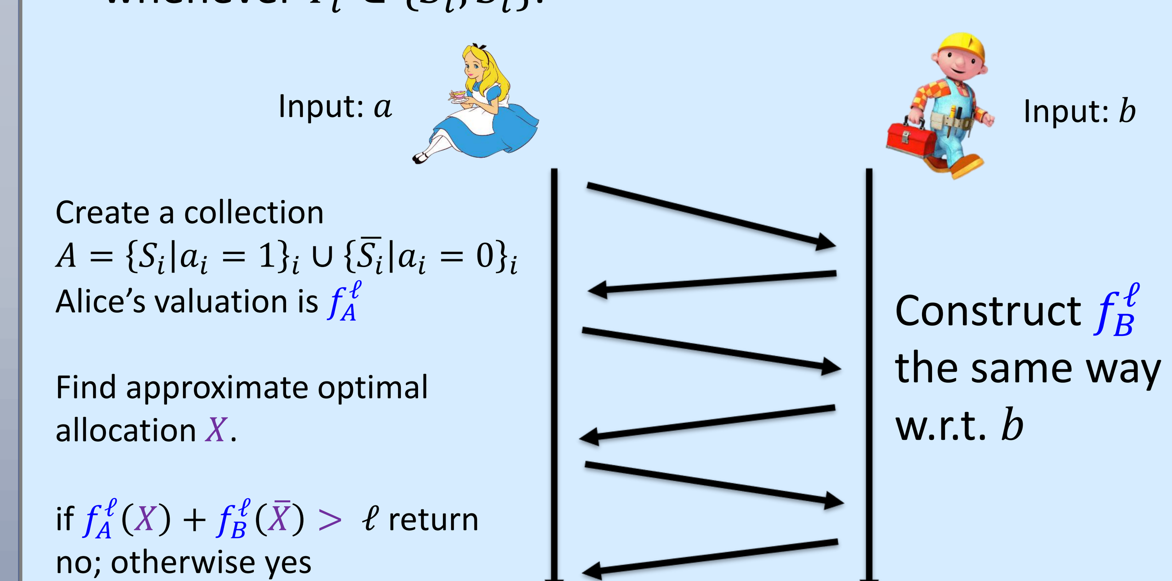
ℓ**-sparse**: ℓ sets are needed to cover $[m]$.

Extremely convenient property (ECP):

$f_S^\ell(T) + f_S^\ell(\bar{T}) = \ell$ for all T

Reduction from Equality

S is ℓ**-independent** if $T = \{T_1, \dots, T_k\}$ is ℓ-sparse whenever $T_i \in \{S_i, \bar{S_i}\}$.



Correctness Proof

$a = b$
⬇
$f_A^\ell = f_B^\ell$
⬇ (ECP)
$f_A^\ell(X) + f_B^\ell(\bar{X}) = \ell$
⬇
output yes

$a \neq b$
⬇
Exists i: $S_i \in A, \bar{S_i} \in B$;
⬇ (ECP)
$f_A^\ell(S_i) + f_B^\ell(\bar{S_i}) = 2\ell - 1$
⬇
$f_A^\ell(X) + f_B^\ell(\bar{X}) > \ell$
⬇
output no

Randomized Protocols

New communication complexity problem

EXIST-FAR-SETS:

Alice and Bob get sets $S_A^1, \dots, S_A^k, S_B^1, \dots, S_B^k$

Return 1 if exists i, such that $|S_A^i \cap S_B^i| \leq 2$

0 if for all i, $|S_A^i \Delta S_B^i| \leq 2$

Claim: **EXIST-FAR-SETS** requires exponential communication.

Definition: $X = \{X_1, \dots, X_k\}, Y = \{Y_1, \dots, Y_k\}$ is

ℓ**-compatible** if:

- $|X_i| = |Y_i| = \frac{m}{2} + 1$ for all i
- For all i Either $|X_i \cap Y_i| \leq 2$ or $|X_i \Delta Y_i| \leq 2$
- X, Y are ℓ-sparse
- For every set $S \subseteq [m]$ of size $< \ell/2$ exist i, j: $S \subseteq X_i, Y_j$

Claim: **EXIST-FAR-SETS** with inputs that satisfy the ℓ-**compatibility** property, requires exponential communication.

Main theorem: approximating welfare is at least as hard as solving **EXIST-FAR-SETS** with inputs that satisfy the ℓ-**compatibility** property

References

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