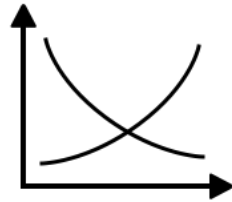


# FAT in Resource Allocation: Equilibrium to the Rescue



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(Based on our paper “Fair Allocation through Competitive Equilibrium from Generic Incomes”)

# Resource Allocation

- **Who** gets **what**
- Fair allocation decisions should take into account:
  1. who **wants** what (**preferences**)
  2. who's **entitled** to what
- Increasingly driven by algorithms

# Motivating Scenario

- Players: 2 food banks catering to populations with different needs
- Resources: Donated food items to allocate among food banks



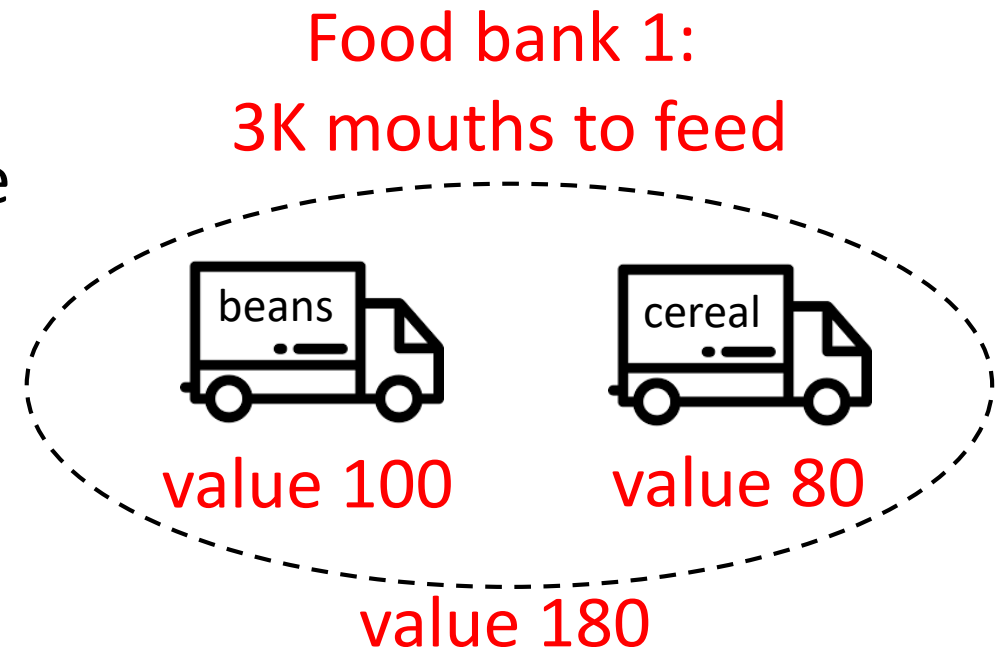
# Motivating Scenario

- Players: 2 food banks catering to populations with different needs
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  1. Using (real) money is inappropriate



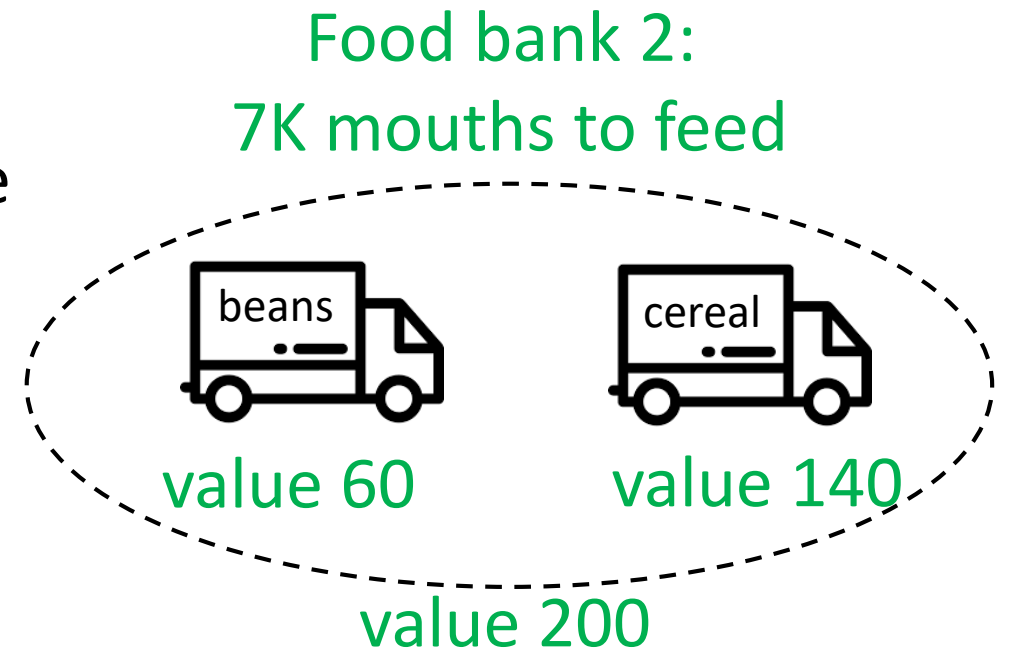
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# Motivating Scenario

- Players: 2 food banks catering to populations with different needs
- Resources: Donated food items to allocate among food banks
- Properties:
  1. Using (real) money is inappropriate
  2. Preferences (roughly) additive
  3. Entitlements by population served
- Main question: What constitutes a **fair allocation** of the items?

# Additional Examples

What constitutes a **fair allocation** of:

- Courses to students?
- Shifts to workers?
- Computational resources across a university/company?
- Heirloom items among family members?



# Fundamental Fairness Notion #1: Fair Share

- Procedure of dividing a cake fairly:
  - first kid **divides** the cake
  - second kid **picks** a piece
- First gets her **fair share** =  $\frac{1}{2}$  her value for the entire cake
  - Second gets at least  $\frac{1}{2}$



\*Image from Spliddit

# Fundamental Fairness Notion #1: Fair Share

- Procedure of dividing a cake fairly:
  - first kid **divides** the cake
  - second kid **picks** a piece
- First gets her **fair share** =  $\frac{1}{2}$  her value for the entire cake
- But we're allocating **indivisible** items (truckloads...)!
  - Seek **as fair** an allocation **as possible**



# Fundamental Fairness Notion #2: Maximin Share

*indivisible items*

- Procedure of dividing ~~a cake~~ fairly:
  - first kid **divides** the cake
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# Fundamental Fairness Notion #2: Maximin Share

*indivisible items*

- Procedure of dividing ~~a cake~~ fairly:
  - first kid **divides** the ~~cake~~ *items into 2 piles*
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*indivisible items*

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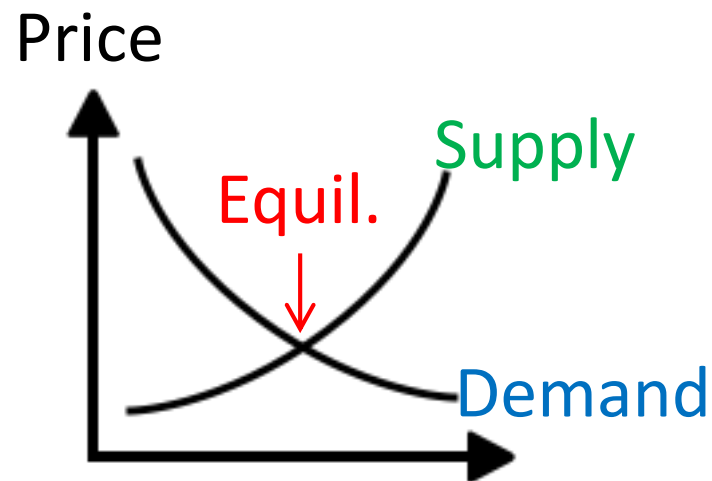
# Fundamental Fairness Notion #2: Maximin Share

- Procedure of dividing indivisible items fairly:
  - first kid **divides** items into 2 piles
  - second kid **picks** a pile
- First gets her **maximin share** = as close to  $\frac{1}{2}$  as possible
- A Rawlsian guarantee from behind a “thin veil of ignorance”



# Equilibrium to the Rescue?

- In market equilibrium **supply=demand**
- Equilibrium **prices** are:
  - sufficiently high (no over-**demand**)
  - sufficiently low (market clears – no excess **supply**)



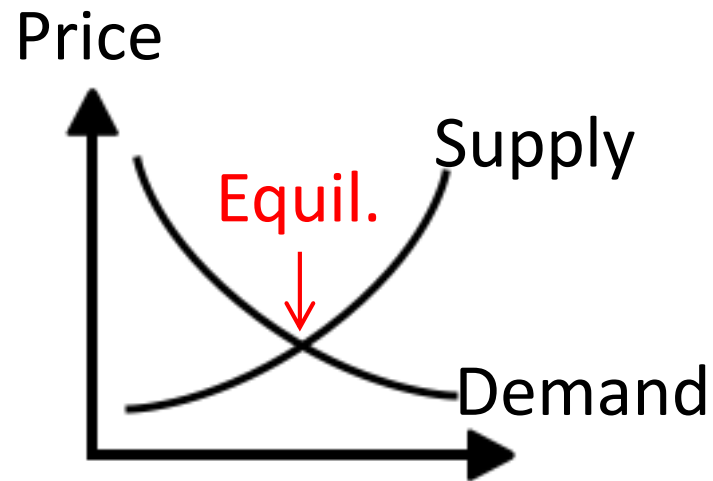
# Equilibrium to the Rescue?

- Market equilibrium applicable to us via fake “money”
- How it works:
- Food banks get **budgets** (say 10K “points”) representing entitlements
- Equilibrium item **prices** are such that
  - each food bank takes its best affordable set of items
  - market clears



# Equilibrium to the Rescue?

- Amazing properties of equilibrium allocation (with equal budgets):
  - Each player gets at least her **maximin share** [Varian'74, Budish'11]
  - **Transparency** – everyone faces same prices
  - Efficiency



# Too Good to Be True?

- Equilibrium not guaranteed to exist!

- Example:

Food bank 1:  
Budget 10K



Food bank 2:  
Budget 10K

- Price **above** 10K → market doesn't clear
- Price **at or below** 10K → over-demand

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  - Price **at or below** 10K → over-demand
- 
- Reason for optimism: Issue is **equal** budgets – a knife's edge case
    - Disappears when entitlements are **slightly different** (as in reality)

# Equilibrium Exists Except on Knife's Edge

Food bank 1: Budget  $10K + 1$   Food bank 2: Budget  $10K$

- Equilibrium price =  $10K + 1$
- We show: With **slightly different** budgets an equilibrium exists

# Equilibrium Exists Except on Knife's Edge

Food bank 1: Budget  $10K + 1$   Food bank 2: Budget  $10K$

- Equilibrium price =  $10K + 1$
- We show: With **slightly different** budgets an equilibrium exists
- Theorem: For 2 additive players with equal budgets, after tiny random perturbations to the budgets, a market equilibrium exists
- [More existence theorems in paper]

# Deriving New Fairness Notions from Equilibrium

- With **slightly** perturbed budgets, is the equilibrium allocation still fair?
- Intuition: fairness will hold **approximately**
  - relaxed to the extent necessary given indivisibilities

# Deriving New Fairness Notions from Equilibrium

- With **slightly** perturbed budgets, is the equilibrium allocation still fair?
- Intuition: fairness will hold **approximately**
  - relaxed to the extent necessary given indivisibilities
- What if the budgets are **very** different?

**Food bank 1:**  
**3K mouths to feed**

**Food bank 2:**  
**7K mouths to feed**

- What constitutes a fair allocation of indivisibles among **unequals**?

# Deriving New Fairness Notions from Equilibrium

- Theorem: In any equilibrium allocation among players with (possibly very different) budgets, each player gets at least her **proportional maximin share** (or as close as possible given indivisibilities)



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    - first **divides** items into 10 piles
    - second **picks** 7 piles
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    - 7K mouths to feed

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<b>3K mouths to feed</b>	<b>7K mouths to feed</b>

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- First gets **proportional maximin share** = as close to 30% as possible

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- [Another fairness notion from equilibrium in paper]

# Take-Aways

- Markets, prices, equilibrium: **useful toolbox** for defining and implementing fairness + transparency
  - alongside “usual suspects” like statistics, ML
- 1. Equil. helps **define** fair allocation of **indivisibles among unequals**
  - Open: **Envy-freeness** notions among unequals
- 2. The **existence** of **approx. fair allocations** follows from equilibrium existence after tiny budget perturbations
  - Open: **Typical** existence

# Thank you!

“Algorithms and Law” initiative –  
please contact me for details!

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