When are Welfare Guarantees Robust?

I-CORE DAY 2016

INBAL TALGAM-COHEN (I-CORE POST-DOC)

BASED ON JOINT WORK WITH TIM ROUGHGARDEN & JAN VONDRAK

The Welfare Maximization Problem

Central problem in AGT; interesting algorithmically

- [e.g., Murota'96, Lehmann-Lehmann-Nisan'06, Vondrak'08, Khot et al.'08, Feige'09…]

Setting:

m items, n buyers, valuation functions $v_1, \ldots, v_n$

v_i(bundle S_i of items) = value of buyer i for S_i

Monotonicity: $v_i(S_i) \leq v_i(T_i)$ whenever $S_i \subset T_i$

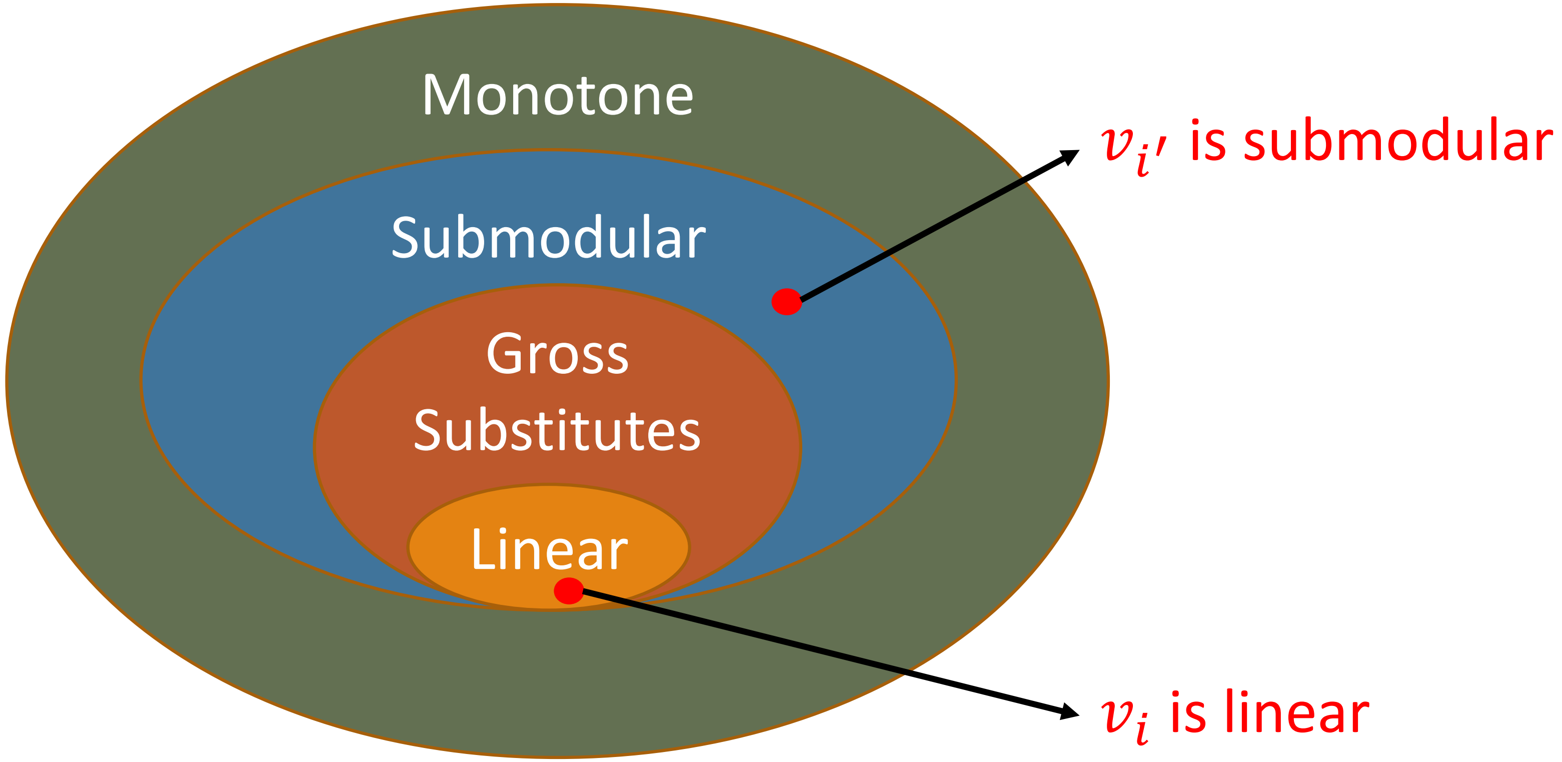
The algorithmic problem:

Given oracle access to the valuations

Output an allocation of items $S_1, \ldots, S_n$

Goal: maximize welfare $\sum_{i \in N} v_i(S_i)$

Hardness Determined by Valuation Class



A Trichotomy

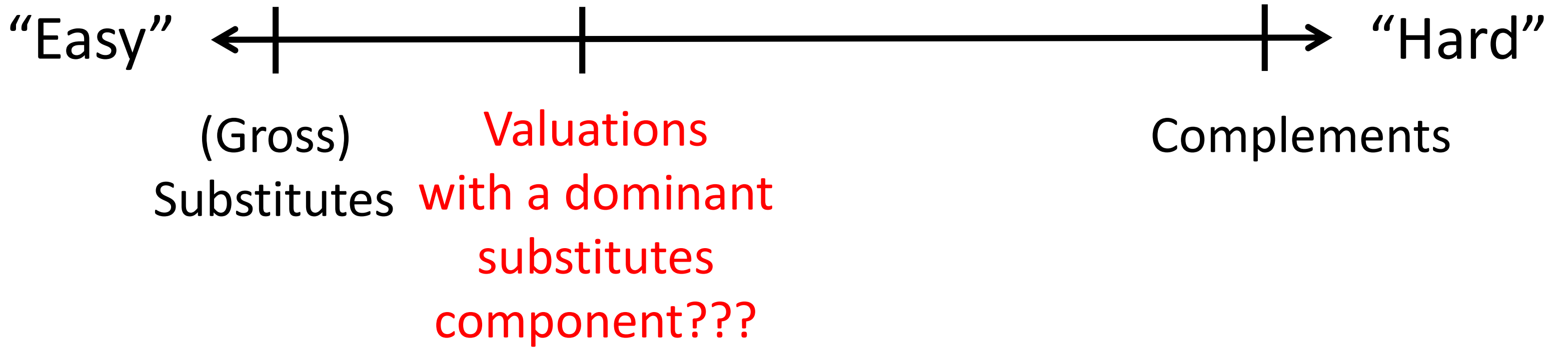
Valuations	Welfare Maximization
Gross substitutes	Easy
Submodular (more generally, complement-free)	Easy to approximate within constant
General	Hard

In Economics

Substitutes (“coffee and tea” rather than “coffee and cream") are a very central but stringent assumption

- [e.g., Kelso-Crawford’82]

For mechanism and market design:



<u>Example supporting folklore belief</u>: Spectrum auctions seem to work…

Research Question

Agenda: Evaluate rigorously whether the main takeaways from the study of gross substitutes remain valid if the substitutes condition holds approximately

This work (ongoing): Evaluate whether the nice algorithmic properties hold

Meta-question: Is algorithmic approximation tied to economic approximation?

Overview of Results

For a natural notion of ϵ-close to gross substitutes:

Negative results in the value query model
- Optimal welfare cannot be approximated, demand queries cannot be simulated

Positive results with
- Additional structure on the valuations
- Demand queries
- Subclasses of gross substitutes

Bottom line: The main take-aways for GS don't hold AS IS. Standard methods may fail. A more delicate story waiting to be uncovered?

Related Work

[Karande-Devanur'07]: For divisible items, "markets do not suddenly become intractable if they slightly violate the weak GS property"

[Singer-Hassidim'16]: Robust welfare maximization for submodular valuations; related techniques

[Lehmann-Lehmann-Nisan'06, Feige et al '14]: Notions of closeness to submodular (and more general) valuations

GS & Approximate GS

Gross Substitutes

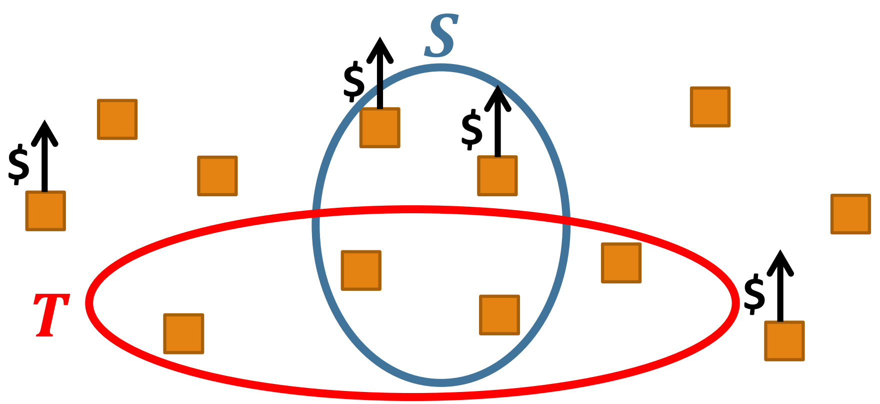
Let $\vec{p} \leq \vec{q}$ be vectors of item prices

A bundle S is in demand given $\vec{p}$ if it maximizes the utility $v_i(S) - p(S)$

v_i is GS if:

- for every S in demand given $\vec{p}$,
- there exists T in demand given $\vec{q}$ which contains every $j \in S$ whose price didn't increase

Seems brittle…



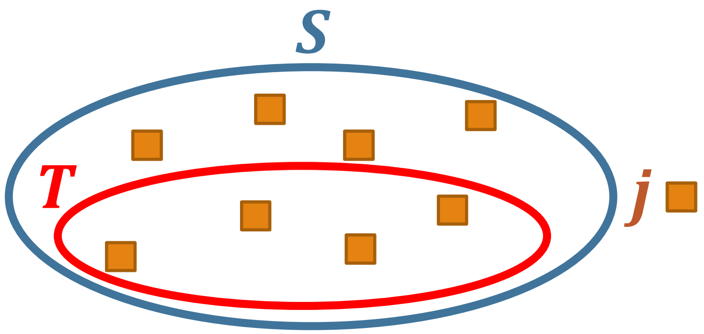
Subclass and Superclass of GS

Subclass:

v_i is linear if there is a vector $\vec{a}$ of item values such that $v_i(S) = c + \sum_{j \in S} a_j$

- Additive if $c = 0$
- Welfare maximization easy

Superclass:



v_i is submodular if its marginal values

$$v_i(j \mid S) = v_i(\{j\} \cup S) - v_i(S)$$

are decreasing

$$v_i(j \mid T) \geq v_i(j \mid S) \text{ whenever } T \subset S$$

- Greedy gives 2-approximation [LLN'06,FNW'78]

Approximate GS

Idea: v is a 2^m-vector of values; add small pointwise perturbations

- Either true valuation is perturbed, or oracle access is erroneous

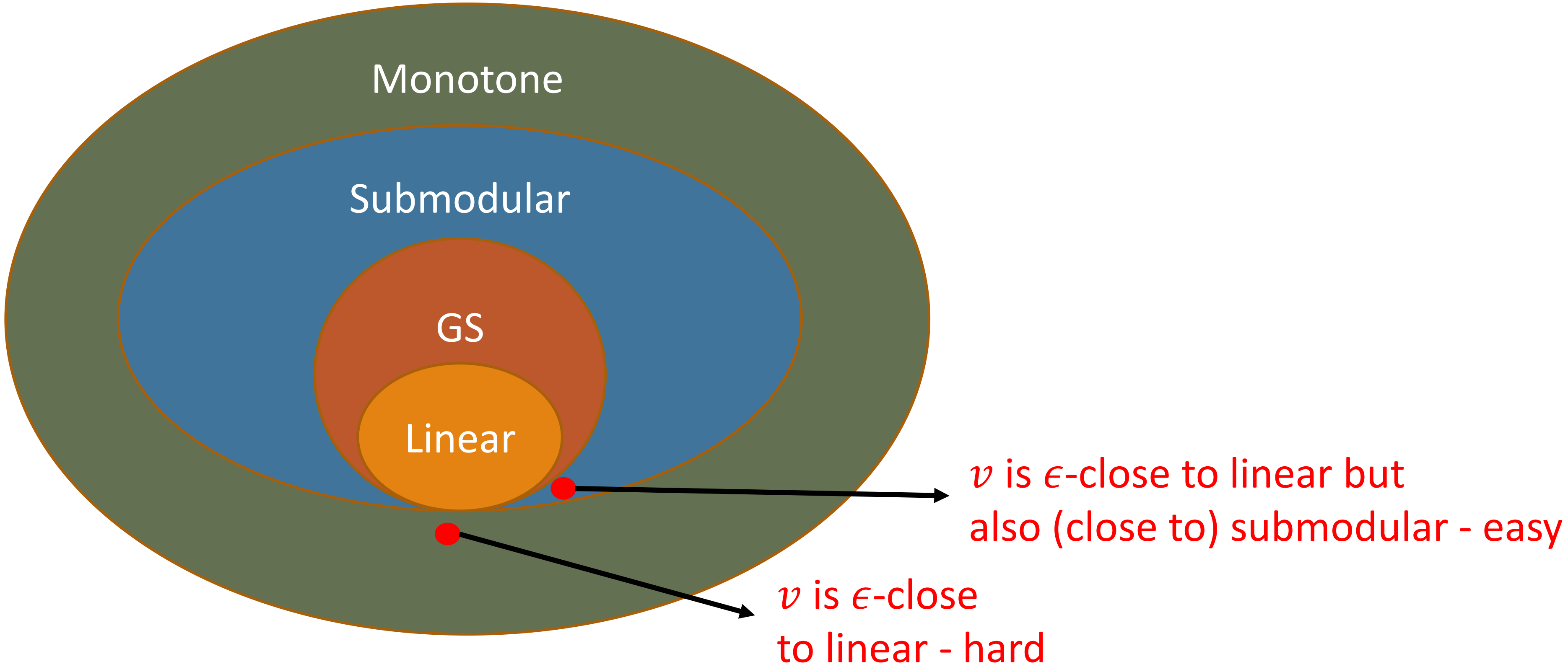
Definition: v is ϵ-close to GS if there is a GS valuation v' such that

$$v'(S) \leq v(S) \leq (1+\epsilon)v'(S) \text{ for every } S$$

Main question, restated: Do the laudable properties of gross substitutes degrade gracefully with ϵ? (e.g. can welfare be approximated up to $(1-\epsilon)$?)

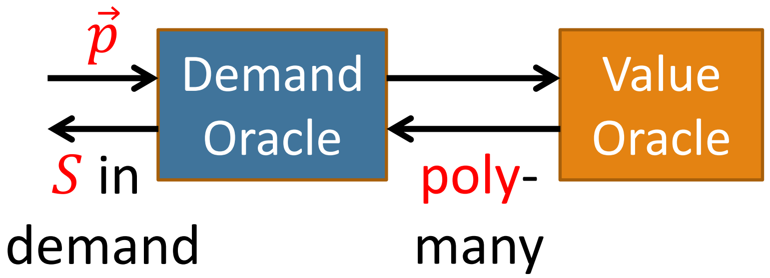
Focus in this talk: ϵ-close to linear

Summary of Answer for Value Queries



Value and Demand Queries

2 types of oracle access to valuations:

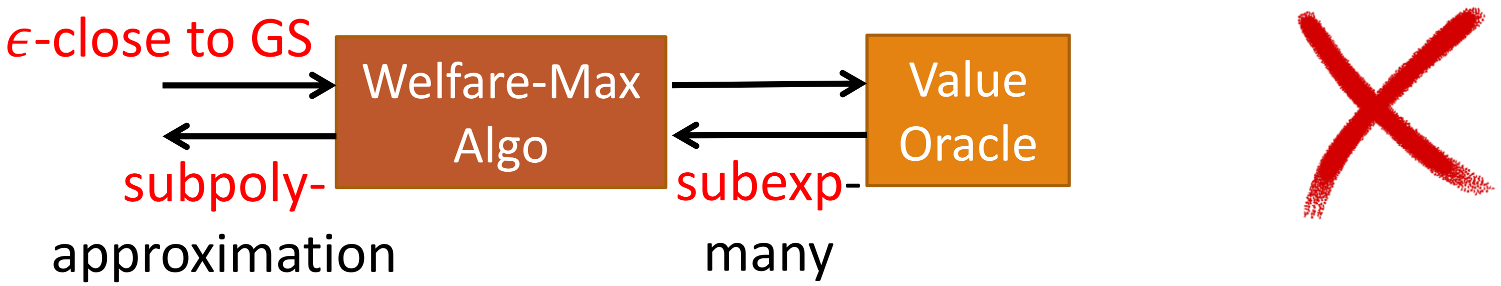


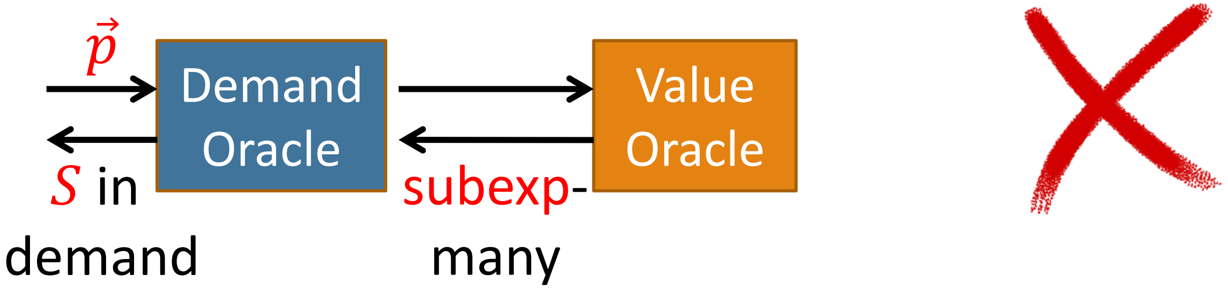
In general: A value query reduces to poly-many demand queries
- [Blumrosen-Nisan]

For GS:
- Welfare can be maximized with poly-many value queries [Murota'96]
- A demand query reduces to poly-many value queries [Bertelsen'04]

Negative Results for Value Queries

For ϵ-close to GS: we show 2 impossibilities





Conclusion: No sweeping generalization of GS properties

What Goes Wrong: Linear vs. Additive

If v is ϵ-close to additive ($v'(S) = \sum_{j \in S} a_j$)

- $a_j \leq v(j) \leq (1+\epsilon)a_j$
- Can recover a_j up to $(1+\epsilon)$

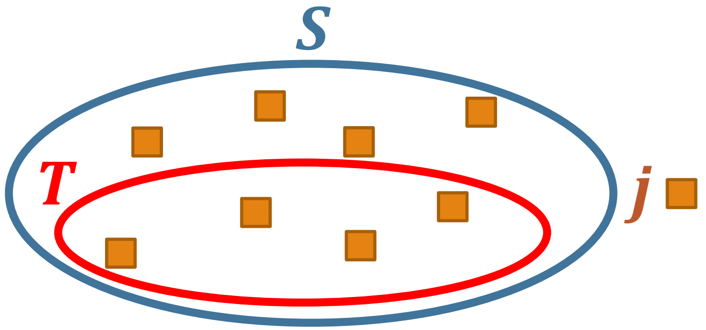
If v is ϵ-close to linear ($v'(S) = c + \sum_{j \in S} a_j$)

- $a_j - \epsilon c \leq v(j \mid \emptyset) \leq (1+\epsilon)a_j + \epsilon c$
- Cannot recover an approximate version of v'
- Information theoretic impossibility

Possible solution: Strengthen notion of approximate GS so marginals are approximated

$\Longrightarrow$ From pointwise approximation to approximation at marginal level

Marginal Closeness and Submodularity



Notation: Marginal $v(j|\cdot)$ maps S to $v(j|S)$

Definition: v is marginal-ϵ-close to decreasing if

$$\forall j:\ v(j|\cdot) \text{ is } \epsilon\text{-close to decreasing}$$

Equivalent for $\alpha = 1 + \epsilon$

Definition [LLN'06]: v is α-submodular

$$\alpha v(j \mid T) \geq v(j \mid S) \text{ whenever } T \subset S$$

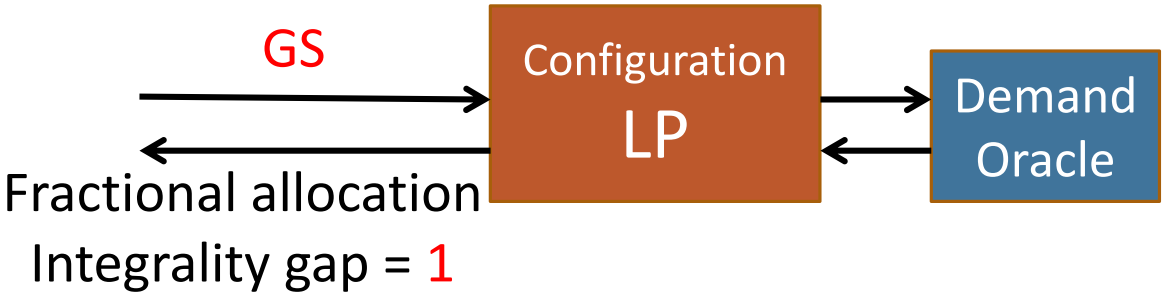
Proposition: For ϵ-close to linear and α-submodular valuations, greedy achieves a $\frac{1-3\epsilon}{\alpha}$-approximation to welfare with value queries in poly-time

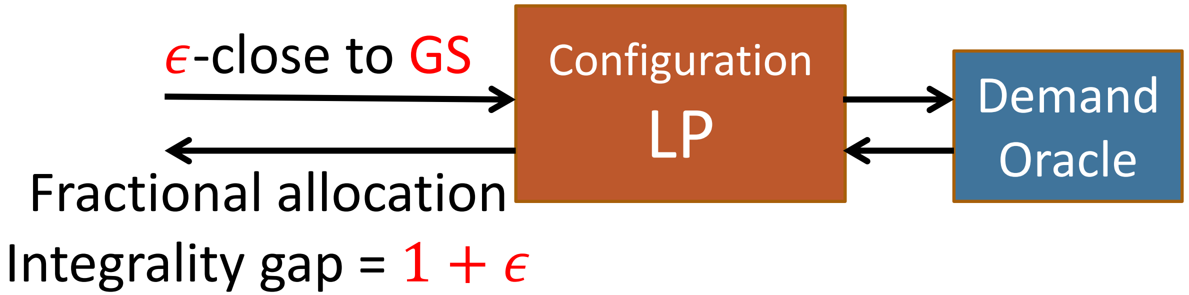
- [compare to Sviridenko et al.'15]

Demand Queries: Positive Results

Optimal welfare can be estimated with demand queries:

<u>Proposition</u>: For every class C of valuations for which the integrality gap of the configuration LP is γ, can estimate welfare within $(1+\epsilon)\gamma$ for ϵ-close to C





Oblivious rounding techniques (linear, XOS) also enable approximation

Performance of Standard Algorithms

Proposition: Approximation guarantee of Kelso-Crawford ascending auction degrades gracefully for ϵ-close to subclasses of GS

In general, can fail miserably…

Conclusion: A Cautionary Tale

Ideally, approximation guarantees degrade gracefully under small deviations

Negative results for ϵ-close to linear with value queries

$\Longrightarrow$ No "generic reason" for belief that "close to substitutes is easy"

Positive results for ϵ-close to linear with additional structure or demand queries

$\Longrightarrow$ It matters how closeness is defined

$\Longrightarrow$ Robust results can require new ideas

The Bigger Picture

[Lehmann-Lehmann-Nisan’06]: “This paper’s main message is that the case of submodular valuation functions should be the focus of interest.”

Led to a lot of groundbreaking research over past decade

It’s a good time to be working on GS (and its approximations)…

[Hsu et al. ‘16; Paes Leme and Wong ’16; Cohen-Addad et al ’16; …]

Open Problems

1) Is it possible to get a more sweeping positive result? E.g. for

- ϵ-close to GS and submodular valuations with value queries
- ϵ-close to GS with a normalization guarantee
- Other natural notions of closeness to GS

2) Do nice economic aspects of GS hold for ϵ-close to GS?

- Possibly need notions of approximate market equilibrium, robust prices, robust auction formats
- [*cf*. Bikhchandani-Ostroy'02, Ben-Zwi et al. '13, Feldman et al.'14, Duetting-Roughgarden-T.C.'14 Roughgarden-T.C.'15]