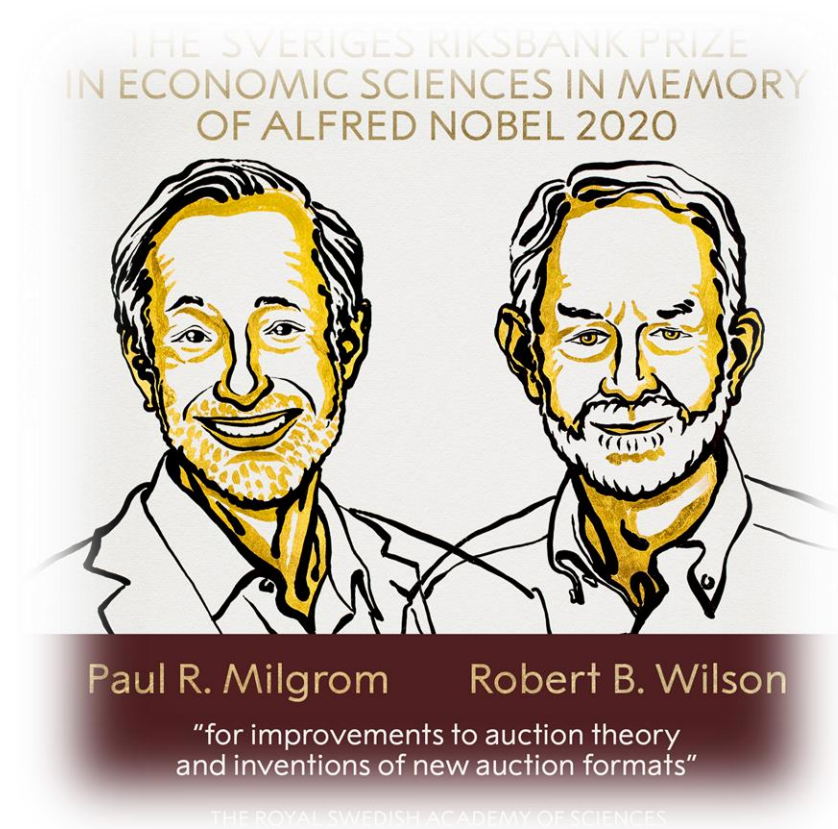


Auctions with Correlated and Interdependent Values

Part I: Contributions of Robert Wilson and Paul Milgrom

presented by **Piotr Dworczak**
(Northwestern University)



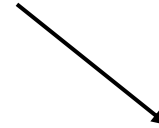
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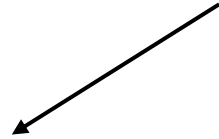
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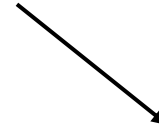
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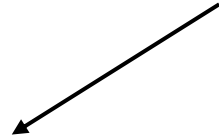
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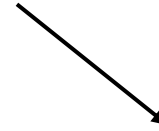
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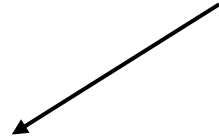


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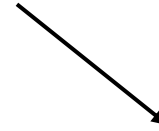
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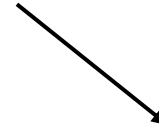
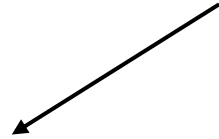
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Common values: Special case when the item is worth the same amount to everyone

Outline of the Tutorial

1. Introduction to the topic (Piotr)

A. Emergence of interdependent-value auctions

B. A real-time common-value auction

C. The framework

2. Foundational papers by Robert Wilson and Paul Milgrom (Inbal)

3. Robustness and auction design (Brendan)

From General Equilibrium to Common-Value Auctions

Classical idea in economics: Market prices **aggregate dispersed information**, thus ensuring efficient allocation of resources (Adam Smith (1776), Lèon Walras (1874) , Friedrich Hayek (1945))

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Black-box approach: These theories are **non-strategic, do not feature dispersed information, and do not explain how prices are found** in practice.

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But: We need a notion of the **true value of the good** which is absent from an IPV model

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In the common-value auction framework, we can ask:

Do equilibrium prices in the auction approximate the true value of the asset?

A real-time common-value auction

1. I will show you an **object with a common value**
2. You should try to assess what that value is (in US \$)
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Random variables X_1, \dots, X_N, S have a known distribution.

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21 563 992,50 PLN \approx 5,828,106.08 USD