Tutorial: Auctions with Correlated and Interdependent Values

Part II: Robustness and Auction Design

PRESENTED BY

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Recall from Part I

Once we move beyond independent private values (IPV), auction format and implementation details matter.

Lesson from [Crémer-McLean '85, '88]: care is needed when designing theoretically "optimal" auctions. Wilson doctrine.

Question: how should we approach the problem of finding good auction formats that discover the "right" outcomes and prices, *robustly*?

Robustness

… in a post-[Crémer-McLean] world.

What do we mean by robustness? Two ideas:

1. Distributional knowledge of the auction
   - Relax the auction's dependence on the type distribution.
     For example: noisy or misspecified prior, sample access, learning.
   - [Fu, Haghpanah, Hartline, Kleinberg '14]: ability to extract the full surplus persists even if distributional knowledge is relaxed.

2. Information available to the bidders
   - Often implicit in the solution concept.
   - Can we relax the assumption that bidders are fully informed about a common prior and strategize with respect to it?

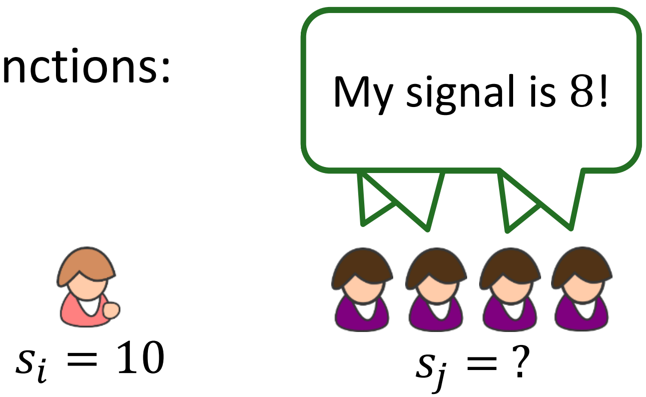
Solution Concepts

Under IPV, it's without loss to restrict attention to direct-revelation auctions that are **truthful in dominant strategies**: each bidder i maximizes utility by declaring her true value, regardless of the other bidders' actions.

What about interdependent values?

Example: 5 bidders, single item, linear value functions:

$$v_i(\boldsymbol{s}) = s_i + \frac{1}{2}\sum_{j \neq i} s_j$$



Truthfulness in dominant strategies:
buyer i wants to report 10 *regardless* of others' true signals.

… even though others' signals can dramatically impact v_i ?!

Solution Concepts

Ex post incentive compatible (IC): for every realization of signals, each bidder maximizes utility by reporting truthfully, if others report truthfully.

Ex post individually rational (IR): for every realization of signals, each bidder gets non-negative utility by reporting truthfully, if others report truthfully.

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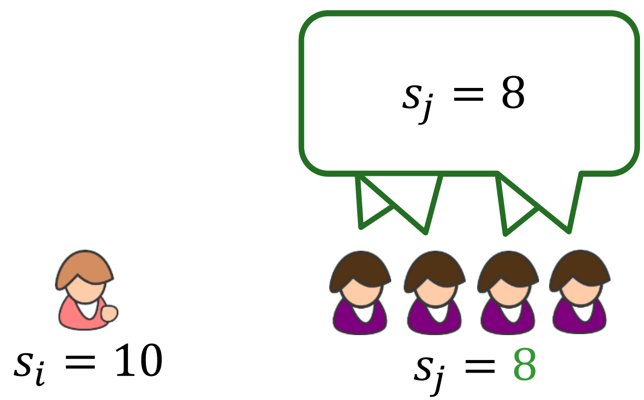

Ex post IC and IR:
buyer i wants to report 10, given that the reports of others are aligned with their signals (and hence informative about v_i).

Solution Concepts

Bayesian incentive compatible: each bidder maximizes her expected utility by reporting truthfully, assuming others report truthfully, where the expectation is over the other bidders' types.

Interim individually rational: each bidder obtains non-negative utility by reporting truthfully, assuming others report truthfully, where the expectation is over the other bidders' types.

Crémer-McLean mechanism: ex post IC, interim IR

- Buyers accept the requested payments because they compare against their anticipated expected utility.
- But some (most) buyers will have negative utility ex post.

Question: what if we insist on ex post IC, ex post IR?

Warmup: Correlated Values

Special case: correlated private values. $v_i(\boldsymbol{s}) = s_i$.

Can we implement the efficient allocation with an auction that is ex post IC and ex post IR?

Vickrey auction:

1. Each bidder reports her value v_i
2. Winner: agent i^* with highest reported value
3. Payment: second-highest reported value

The Vickrey auction is ex post IC and ex post IR (in fact, truthful in dominant strategies) and implements the efficient allocation.

⇒ Robust to correlations between values.

Revenue for Correlated Values

Special case: correlated private values. $v_i(\boldsymbol{s}) = s_i$.

Lookahead auction [Ronen 2001]:

1. Each bidder reports her value v_i
2. Provisional winner i^*: agent with highest reported value
3. Let p be the revenue-optimal price for distribution of v_{i^*}, given other reported values and that v_{i^*} is largest.
4. If $v_{i^*} \geq p$, agent i^* wins and pays p. Otherwise no winner.

Example:

	1	3	4
1	1/6	1/12	1/12
3	1/12	1/6	1/12
4	1/12	1/12	1/6

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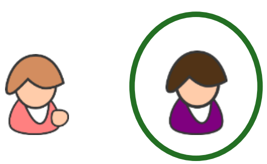

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$p = 1$: Revenue $= 1 \times 1$

$p = 3$: Revenue $= 3 \times 1/2$

$p = 4$: Revenue $= 4 \times 1/4$

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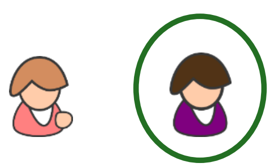

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Properties of Lookahead Auction

Theorem: The expected revenue of the lookahead auction is at least half of the expected revenue of any ex post IC and IR auction.

Proof:

Optimal revenue $\leq R_1 + R_2$, where

R_1: max expected revenue any ex post IC and IR auction can raise from the highest-valued agent only.

R_2: max expected revenue any ex post IC and IR auction can raise from everyone else.

$Rev_{Lookahead} \geq R_1$, from our choice of price p.

Also, by ex post IR, $R_2 \leq E[\max_{j \neq i^*} v_j]$. But the Lookahead auction can always choose $p = \max_{j \neq i^*} v_j$ and sell for sure! So $Rev_{Lookahead} \geq E[\max_{j \neq i^*} v_j] \geq R_2$.

Therefore $2Rev_{Lookahead} \geq R_1 + R_2 \geq$ Optimal Revenue. ∎

Properties of Lookahead Auction

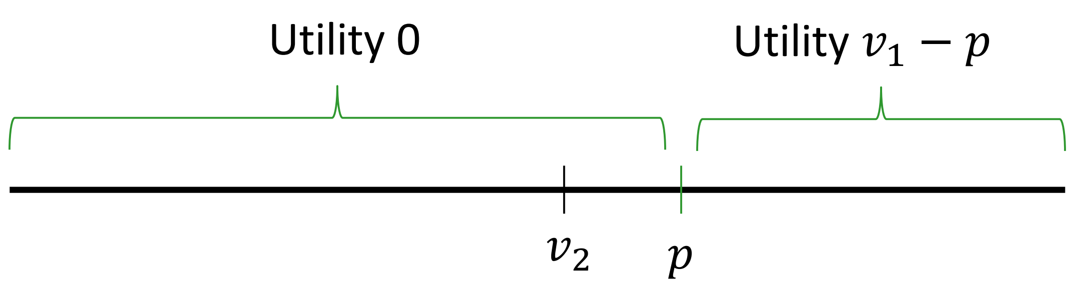
Theorem: The expected revenue of the lookahead auction is at least half of the expected revenue of any ex post IC and IR auction.

Theorem: The lookahead auction is ex post IC and ex post IR.

Why?

Ex post IR since we offer a take-it-or-leave-it price.

Ex post IC because offered price is independent of winner's bid:



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Discussion: a simple argument that is highly robust to the information structure of the agents.

- Similar arguments have been used to design "simple" auctions for multidimensional types. See [Chawla, Fu, Karlin 14].
- Can improve the factor 2 with variations on the lookahead auction: [Dobzinksi, Fu, Kleinberg 2011], [Chen Hu Lu Wang 2011], [Chawla, Fu, Karlin 2014].

Beyond Correlated Private Values

Return to general interdependent values. Maximize welfare?

Generalized Vickrey Auction

1. Each bidder reports her signal s_i
2. Use reported signals to calculate values $v_i(\boldsymbol{s})$
3. Winner: agent i^* with highest value $v_{i^*}(\boldsymbol{s})$
4. Winner i^* pays ? ? ?

What can go wrong? Consider the following example:

$$v_i(\boldsymbol{s}) = s_i + 2\sum_{j \neq i} s_j$$

As s_i increases, it's only *less* likely that agent i will win. Incentive to declare 0.

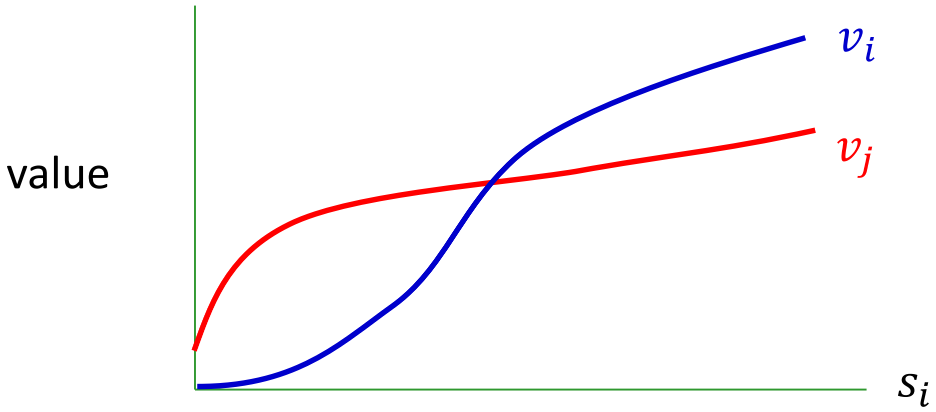
Single-Crossing Condition

Single-crossing condition (for values):

if $v_i(s_i, s_{-i}) \geq v_j(s_i, s_{-i})$,

then for all $s_i' > s_i$

we must have $v_i(s_i', s_{-i}) > v_j(s_i', s_{-i})$.



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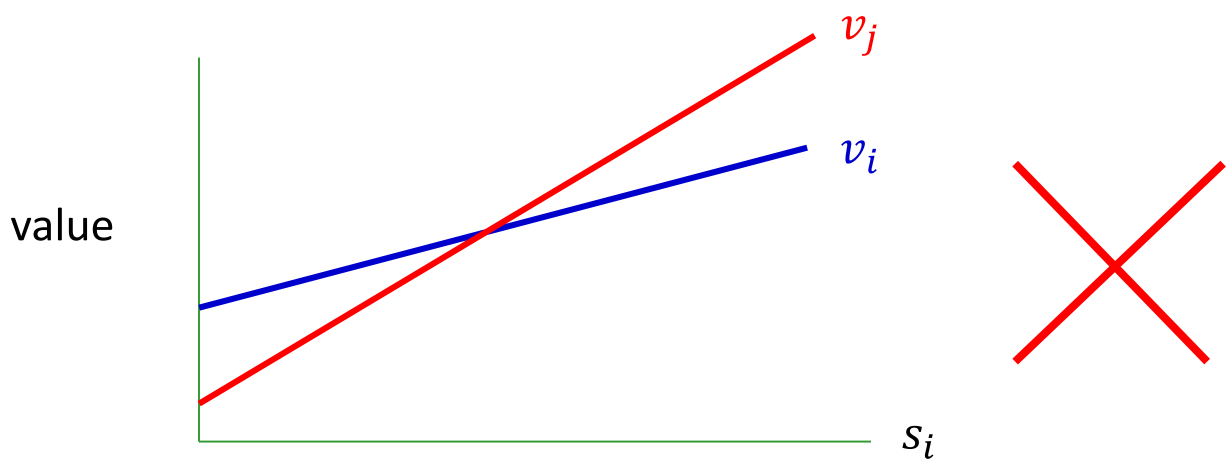


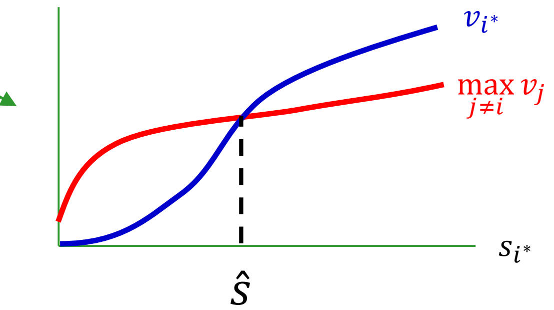
$$v_i(s) = s_i + 2\sum_{j \neq i} s_j$$

Interdependent Values

Assume single-crossing condition holds.

Generalized Vickrey Auction

1. Each agent reports their signal s_i
2. Use reported signals to calculate values $v_i(\boldsymbol{s})$
3. Highest-value agent i^* wins the item
4. Winner i^* pays $v_{i^*}(\hat{s}, s_{-i})$



Theorem: Under the single-crossing condition, the generalized Vickrey auction is ex post IC, ex post IR, and generates efficient outcomes.

What about revenue?

Apply same idea as Lookahead auction: run generalized Vickrey auction, but impose an optimal reserve price on the provisional winner.

Constant approximation to optimal revenue, assuming single-crossing and some additional technical assumptions. [Li 2013, Chawla Fu Karlin 2014]

Another approach:

Assume single-crossing, then characterize the payments that make an allocation rule ex post IC. Express revenue as a form of virtual surplus, similar to [Myerson 1981] in the IPV setting.

Leads to auction formats with optimal or approx. optimal revenue under various conditions; see [Roughgarden and Talgam-Cohen 2013, 2016].

Discussion

1. Many of these results generalize beyond single-item auctions, to richer single-dimensional auction problems. E.g., selling k identical items.

2. Practical implementations
   - Depending on the signal structure, the direct-revelation implementation could be problematic!

   - For symmetric affiliated values [Milgrom Weber 82], generalized VCG can be implemented as an English auction.
   - Can even implement lazy reserves by offering a final take-it-or-leave-it price to the winner of the English auction.
     See [Lopomo 2000], [Chung and Ely 2007], [Li 2013, 2018].

Where to next?

1. Robustness and bidder information
   - Other solution concepts beyond interim and ex post IC?
   - The impact of different information structures on auctions.

2. Beyond single-dimensional settings
   - Significant gaps between theory and practice.

   - Theory: can no longer rely on single-crossing conditions. Many negative results suggest new ideas are required.

   - Practice: can use insights about ascending auctions to design useful, practical combinatorial auctions. E.g., simultaneous ascending auctions for FCC spectrum.

Workshop Schedule

11:00am - 11:10am: Welcome and Opening Remarks

11:10am - 1:00pm: Tutorial - Auctions with Correlated and Interdependent Valuations

11:10am - 12:00pm: Part 1 (**Inbal Talgam-Cohen**, **Piotr Dworczak**)
12:00pm - 12:20pm: Break
12:20pm - 1:00pm: Part 2 (**Brendan Lucier**)

1:00pm - 2:30pm: Social Break

1:45pm - 2:15pm: Student Fireside Chat with **Paul Milgrom** and **Robert Wilson** (location: fireside space)

2:30pm - 5:00pm: Invited Talks

2:30pm - 3:00pm: **Paul Klemperer** - Geometry and Auctions
3:00pm - 3:30pm: **Kira Goldner** - Interdependent Valuations: Beyond Single-Crossing and Single-Item

3:30pm - 3:45pm: Break

3:45pm - 4:15pm: **Songzi Du** - Informationally Robust Auction Design
4:15pm - 4:45pm: **Vincent Conitzer** - Automated Mechanism Design for Correlated Valuations

4:45pm - 5:00pm: **Susan Athey** - Concluding Remarks

5:00pm - 6:00pm: Closing Reception