Price of Anarchy of Simple Auctions with Interdependent Values

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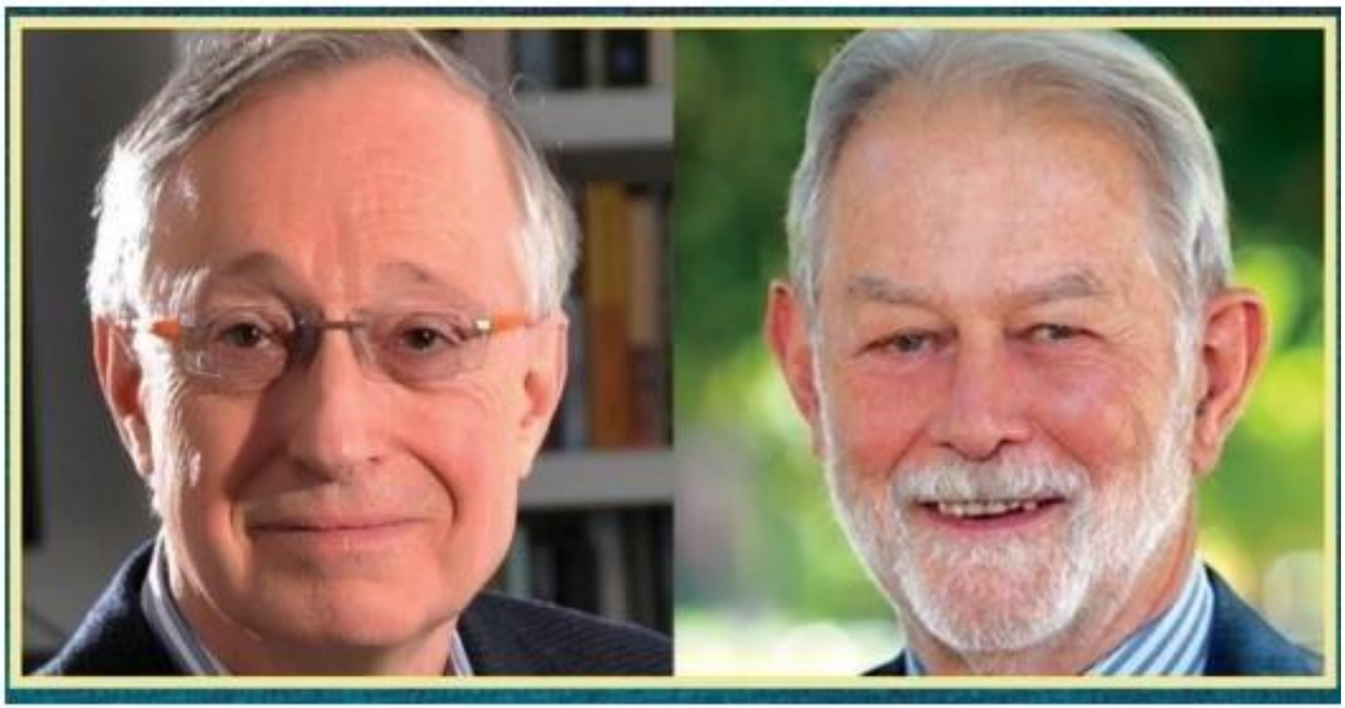
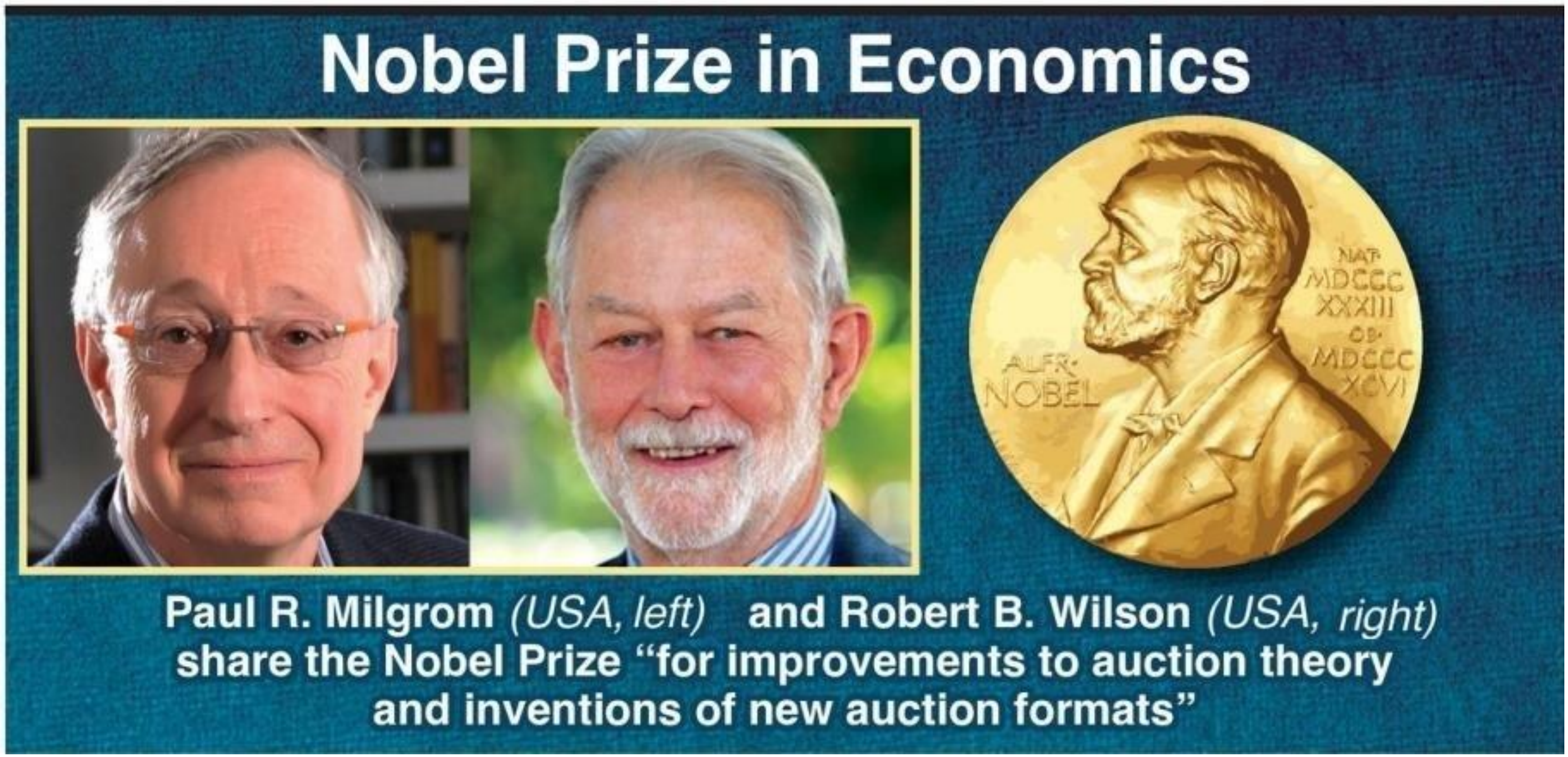
TECHNION

Who are these people?



Nobel prize


Nobel Prize in Economics
NAT·
MDCCC
XXXIII
OB·
MDCCC
XCVI
ALFR·
NOBEL
Paul R. Milgrom (USA, left) and Robert B. Wilson (USA, right)
share the Nobel Prize "for improvements to auction theory
and inventions of new auction formats"

Overview

- Introduction
  - Interdependent Values Background
  - Mechanisms Background
  - Price of Anarchy Background
- Our Results
  - Single item
  - Multiple items
    - Many buyers regime
    - Many items regime
- Conclusion

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Interdependent Values

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Parking

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Cleanliness

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Playgrounds

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Hazards

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Interdependent Values Model
[Milgrom and Weber, 1982]

1 seller, n buyers

Buyer i has a **private signal $\boldsymbol{s_i}$**.

Signals are drawn from a public joint distribution, $\boldsymbol{s} \sim \boldsymbol{F}$

Buyer i's **public valuation function $\boldsymbol{v_i}(\cdot)$** depends on all buyers' signals:

$$\boldsymbol{v_i}(\boldsymbol{s_1}, \boldsymbol{s_2}, \dots, \boldsymbol{s_n})$$

Monotone non-decreasing.

E.g., $v_i(s_1, s_2, \dots, s_n) = \frac{\sum_j s_j}{n}$

Common values [Wilson, '69]

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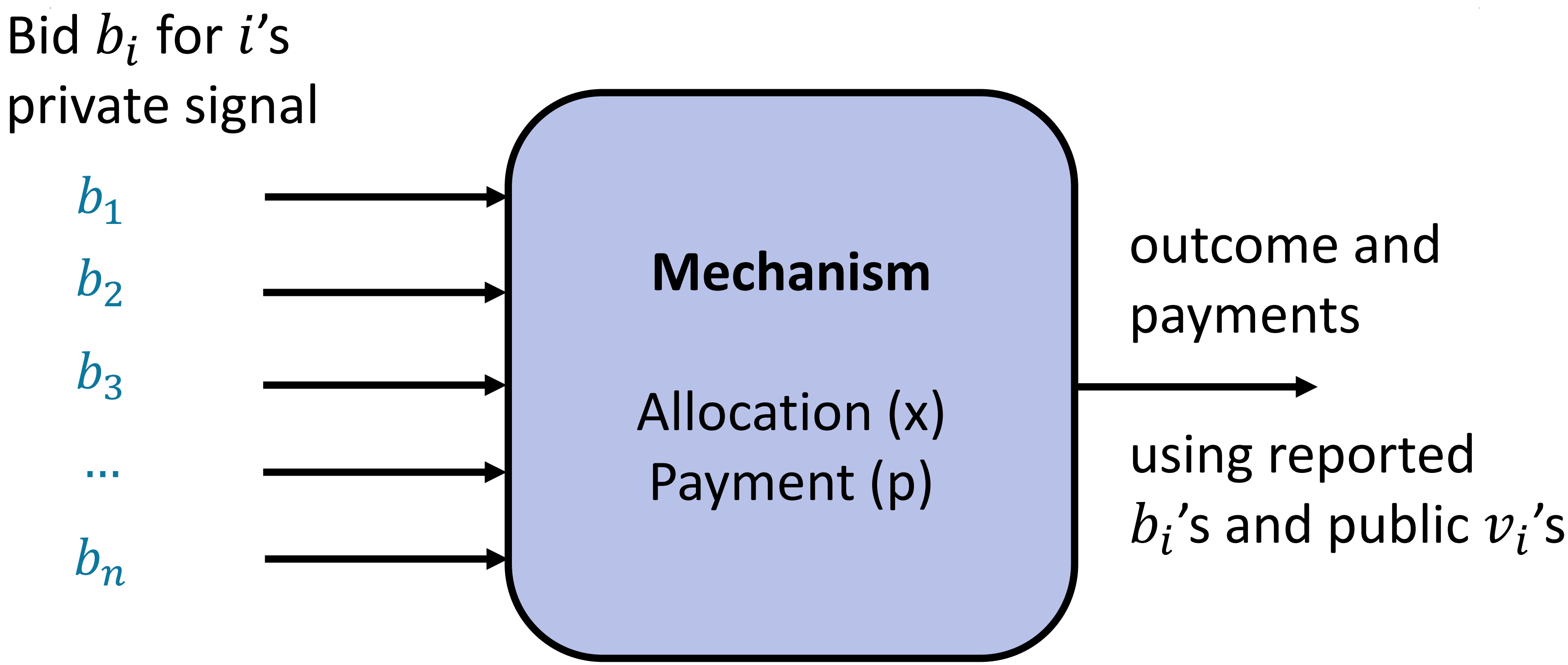
Mechanisms
in interdependent setting



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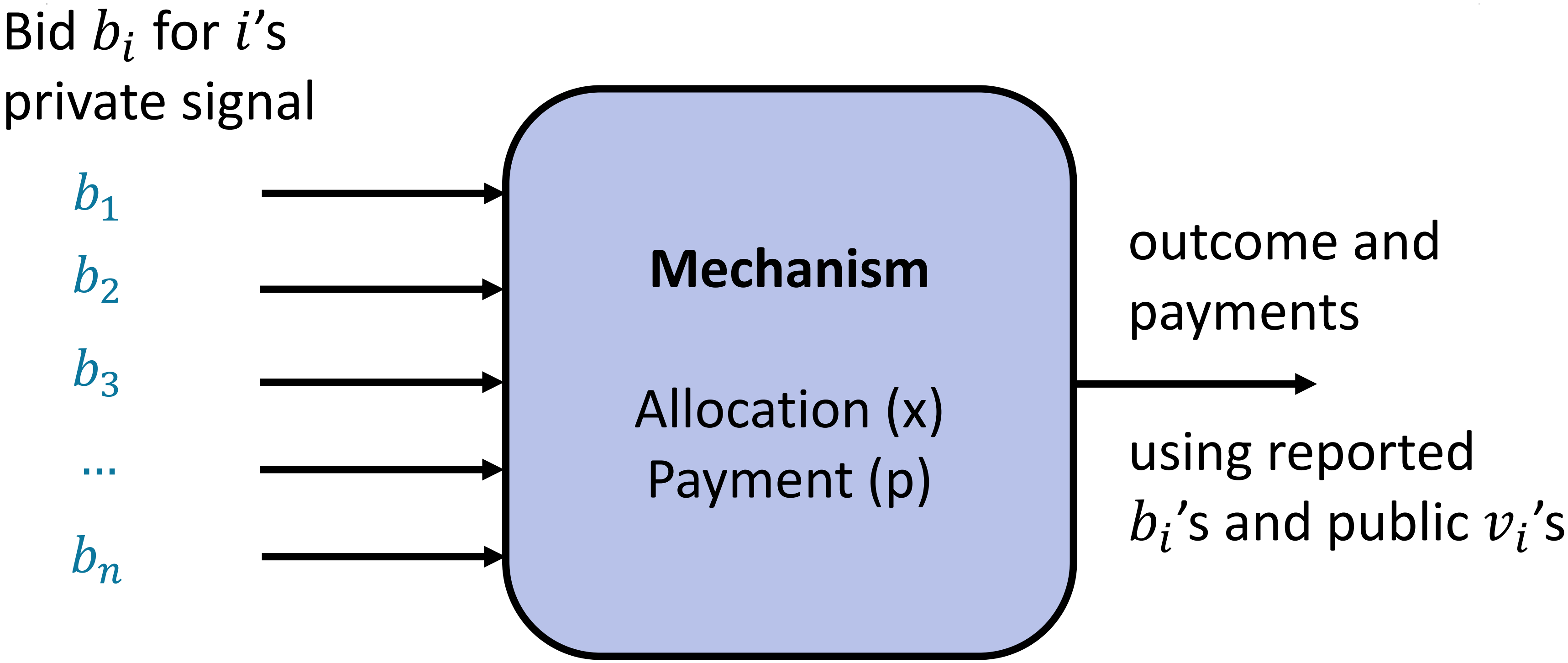
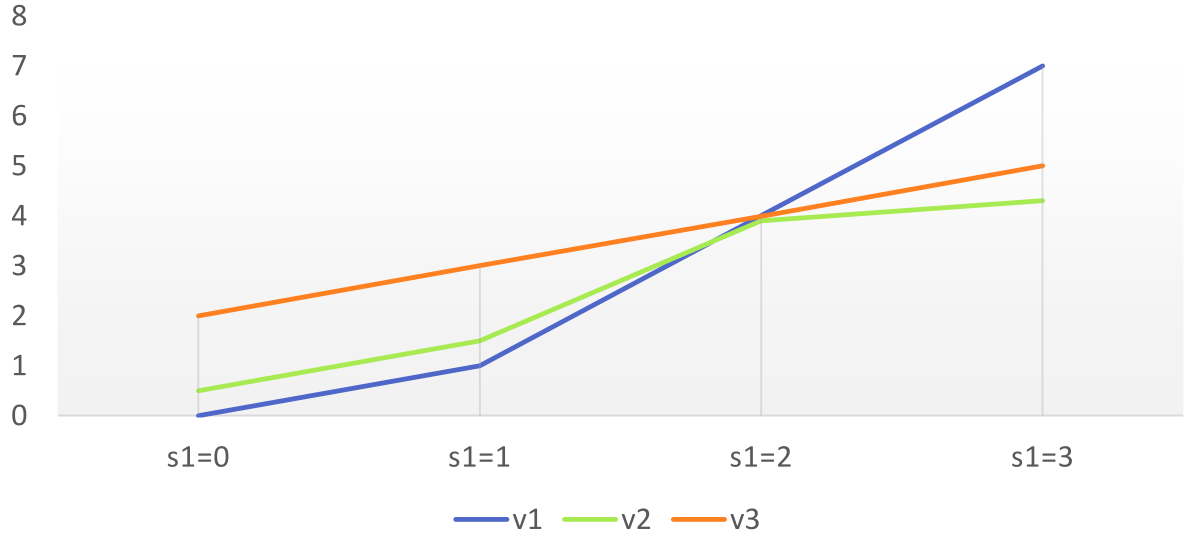


The utility of a player: $u_i(\boldsymbol{b}) = x_i(\boldsymbol{b}) \cdot v_i(\boldsymbol{s}) - p_i(\boldsymbol{b})$

Generalized Vickrey auction (GVA)
[Maskin, 1996, Ausubel, 2000]

- Solicits bids, **b** (**signals**)



Generalized Vickrey auction (GVA)

[Maskin, 1996, Ausubel, 2000]

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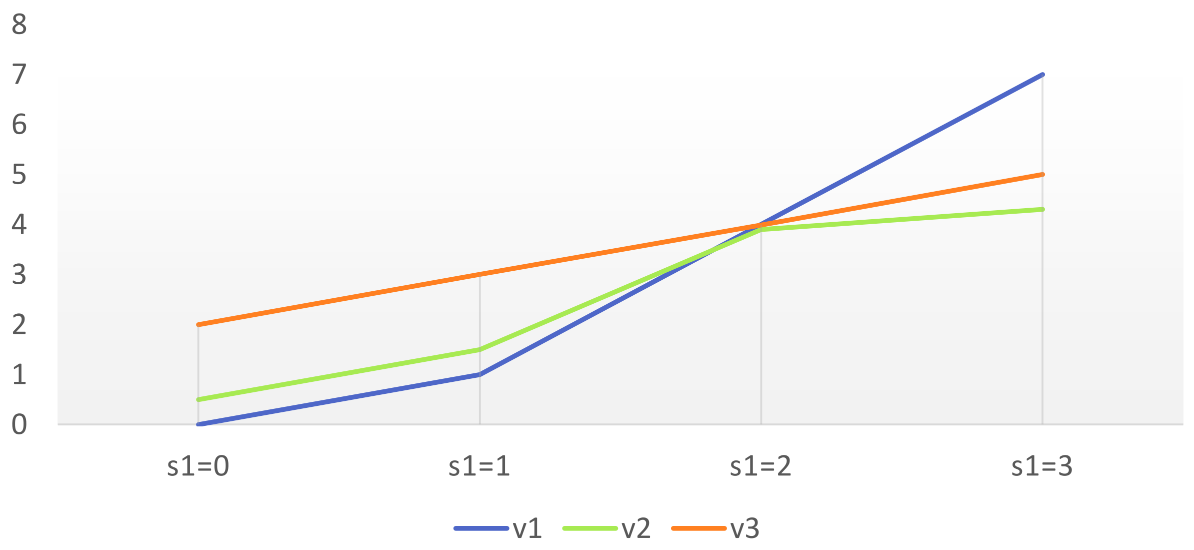
- <u>Allocation</u> – highest value

$$x(b) = \underset{i}{\operatorname{argmax}}\, v_i(\boldsymbol{b})$$



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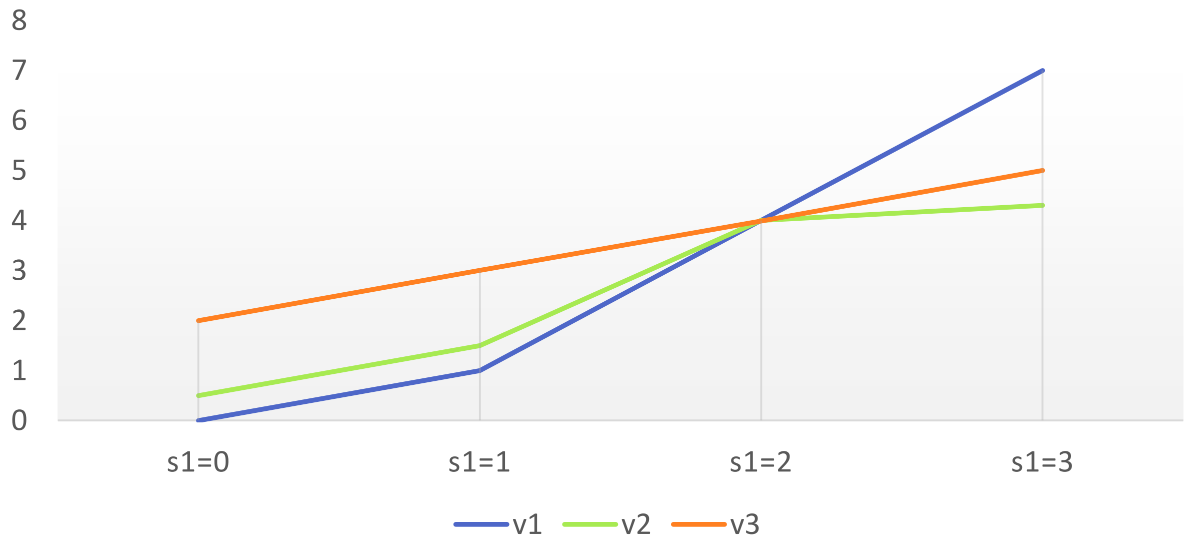

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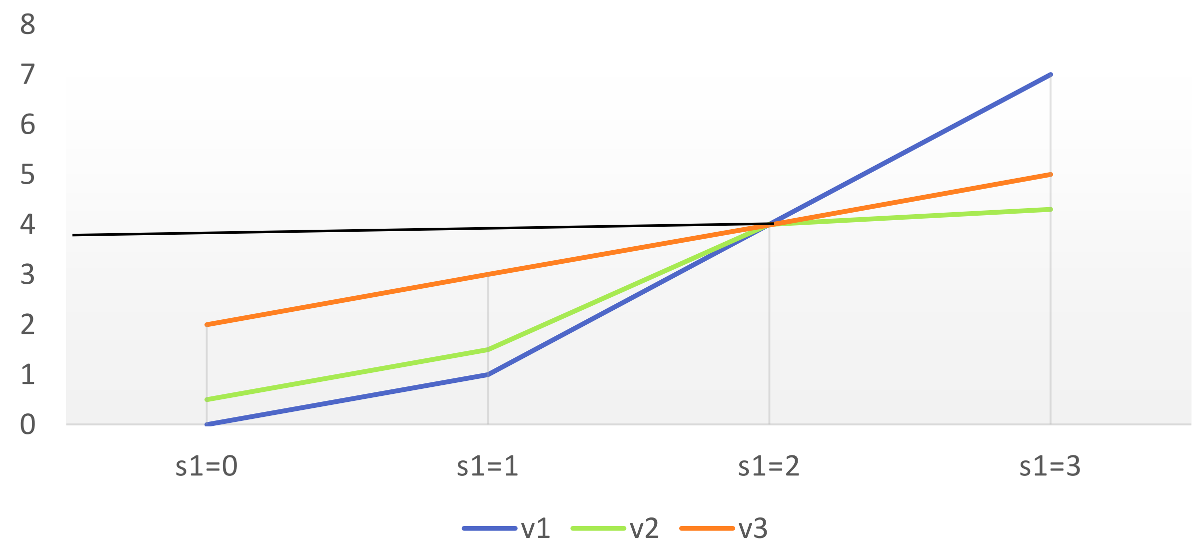

How to evaluate mechanisms?

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- **Social welfare (SW)** - $\sum_i v_{ix_i(\boldsymbol{b})}(\boldsymbol{s})$
- **Optimal welfare** – SW according to optimal allocation

How to evaluate mechanisms?

- Truthfulness

$$u_i(s_i, \boldsymbol{b}_{-i}) \geq u_i(\boldsymbol{b}), \qquad \forall \boldsymbol{b}$$

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  - Mostly requires Single-crossing assumption

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  - Impractical in many cases

[Dobzinski, 2011, Ausubel and Milgrom, 2006, Li, 2017]

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- Price of Anarchy

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Price of Anarchy (PoA)

- The performance of a (not necessarily truthful) mechanism is evaluated using the Price of Anarchy (PoA) measure.
- Equilibrium – no buyer can increase utility by unilateral deviation. (i.e. $\forall b_i'$: $\quad u_i(\boldsymbol{b}) \geq u_i(b_i', \boldsymbol{b}_{-i})$)

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- Incomplete information model $(\boldsymbol{s} \sim \boldsymbol{F})$
- A lot of research on simple mechanisms under independent private values (e.g. simultaneous item auctions)

[Christodoulou, Kovács and Shapira, 2016, Hassidim et al.,2011, Bhawalkar and Roughgarden, 2011, Syrgkanis and Tardos, 2013, Feldman et al., 2013]

Price of Anarchy (PoA)

- A standard tool for Price of Anarchy results– **smoothness framework** [Roughgarden, 2015]:
  - Use the equilibrium hypothesis with respect to an appropriate hypothetical deviation
  - Conclude Price of Anarchy guarantees

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Research Question

Are there Simple Mechanisms with good Price of Anarchy guarantees for Interdependent Values?

Example (interdependent values)

- The Generalized Vickrey auction:
  - Allocation – highest value
  - Payment – value at critical bid
- A single item, n buyers
- Valuation profile:

$$v_1 = \sum s_i, \quad v_2 = 3s_2, \quad v_i = s_i \; (\forall i \geq 2)$$

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Claim: $\boldsymbol{b}$ is an equilibrium$\Rightarrow$ $PoA(GVA) = O(n)$

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- This example is very "asymmetric".

Gamma-heterogeneity

- Definition: a valuation profile is γ-heterogeneous if for every three agents i, j, j', signal profile $\boldsymbol{s}$ and $\delta > 0$:

$$v_j(s_i + \delta, s_{-i}) - v_j(s_i, s_{-i}) \geq \boldsymbol{\gamma}(v_{j'}(s_i + \delta, s_{-i}) - v_{j'}(s_i, s_{-i}))$$

  - E.g., in previous example $\gamma = \infty$

Our Results

	Single Items	Multiple items, Many buyers	Multiple items, Many items
Standard Assumptions	O(n)	O(n)	

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All of our results extends to the c-Single crossing assumption
[Eden et al., 2019]

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Single item - Positive Result

Main Results for Single item:

Under γ-heterogeneity and standard assumptions,
$PoA(GVA) = \gamma$

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Single Crossing(SC)
[Ausubel, 2000, Maskin, 1996, Chawala, Fu and Karlin, 2014]

No-overbidding
[e.g. Bhawalkar and Roughgarden, 2011]

- **Remark** – this result extends to settings with incomplete information (Bayesian-PoA).

Single item - Positive Result

Main Results for Single item:

Under γ-heterogeneity and standard **(general)** assumptions, $PoA(GVA) = \max\{\gamma, c\} + 1$

c-Single Crossing(c-SC)
[Eden et al.2018]
No-overbidding
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c-SC: $c \cdot v_i(s_i + \delta, \boldsymbol{s}_{-i}) - v_i(\boldsymbol{s}) \geq v_j(s_i + \delta, \boldsymbol{s}_{-i}) - v_j(\boldsymbol{s}); \ \forall j$

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Single item - Positive Result

Main Results for Single item:

The previous results are essentially **tight**:

There exists single item settings satisfying c-SC and γ-heterogeneity such that the PoA is **arbitrarily close** to $\max\{\gamma, c\}$.

Single item - Positive Result

Main Results for Single item:

Under γ-heterogeneity and standard **(general)** assumptions, $PoA(\mathbf{2PA}) = \max\{\gamma, c\} + 1$

c-Single Crossing(c-SC)
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- **Remark** – this result extends to settings with incomplete information (Bayesian-PoA).
- This result is **tight** as well.

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Single item – Our Technique

- **Key lemma:** for every two buyers i, j ,signal profile s and $\delta = (\delta_1, \ldots, \delta_{i-1}, 0, \delta_{i+1}, \delta_n) \geq 0$:

$$\underbrace{v_j(\boldsymbol{s} + \delta) - v_j(\boldsymbol{s})}_{\text{Change in j's value under signal increasing}} \geq \Big(\underbrace{v_i(\boldsymbol{s} + \delta) - v_i(\boldsymbol{s})}_{\text{Change in i's value under signal increasing}}\Big) / \max\{\gamma, c\}$$

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- Use a variant of the **smoothness argument** which we adjust to the interdependent values settings
  - hypothetical deviation: bid true signal, s_i
  - Prove a lower bound on the utility under the deviation, using the **key lemma**

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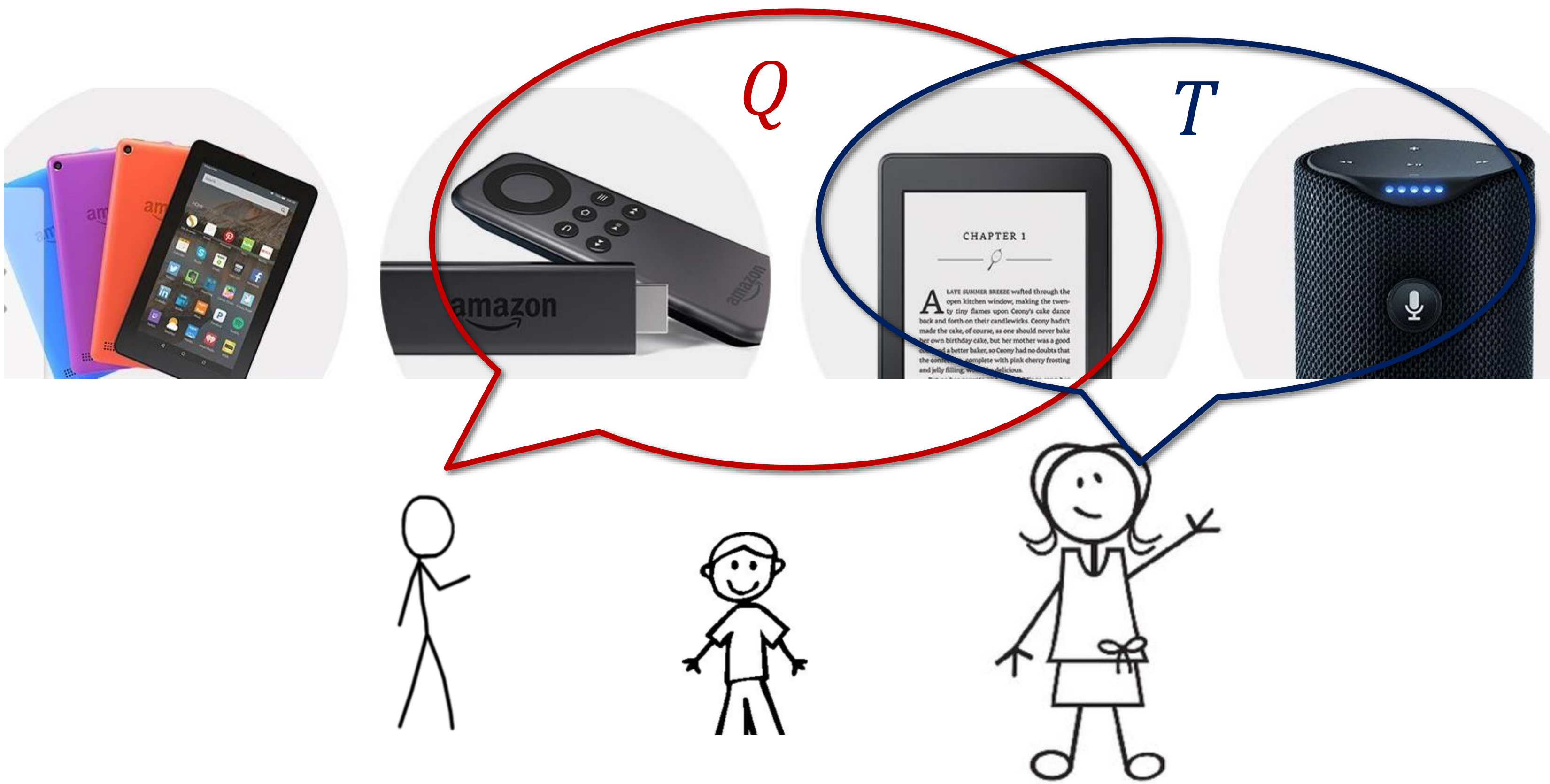
Beyond Single Item

Multiple items

m items, n buyers



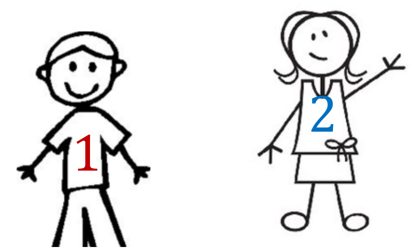
Multiple items, Interdependent values

Buyer i:

Multiple items, Interdependent values

Buyer i:

private signal



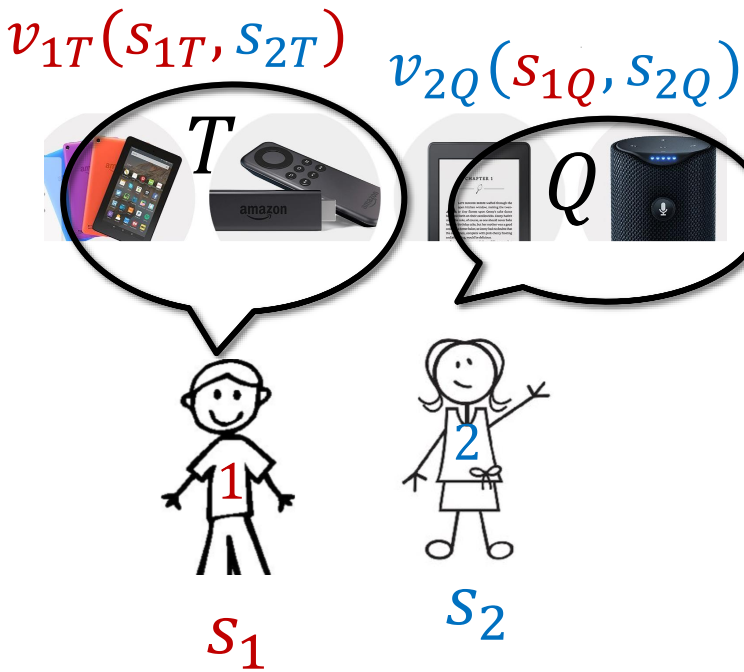
s_1 s_2

Multiple items, Interdependent values

Buyer i:

private signal

public $v_{iT}(\cdot)$ for every set T



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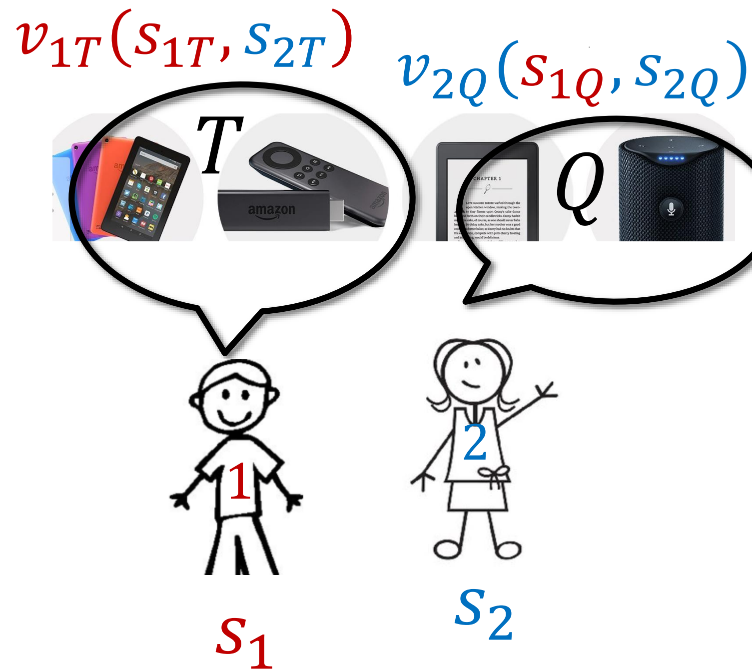

Unit-demand: $v_{ij}(s_{1j}, \ldots, s_{nj})$ for every item j

$$v_{iT}(\mathbf{s}) = \max_{j \in T} v_{ij}(s_{1j}, \ldots, s_{nj})$$

Multiple items

Results separate into two domains: many buyers and many items

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  - Bidders can express their willingness to participate in an auction for every item along with their bid.

Multiple items – Positive result

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  - $n \geq m$
  - **Simultaneous item auctions** – each item is sold separately
  - Bidders can express their willingness to participate in an auction for every item along with their bid.
  - Cannot achieve bounds for incomplete information setting using only the above assumptions.

Multiple items – Positive result

- **Many buyers:**
  - Definitions:

Multiple items – Positive result

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  - Definitions:
    - The truncated value of an agent i, $\tilde{v}_{il}$:

$$\tilde{v}_{il} = \min_{j \neq i} v_{il}(s_{-j}, 0_j)$$

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$$\tilde{v}_{il} = \min_{j \neq i} v_{il}(s_{-j}, 0_j)$$

    - The truncated welfare - $\widetilde{OPT}$ is the optimal welfare with respect to the truncated values.
    - A valuation profile has d-limited knowledge asymmetry if there exists a constant d such that

$$d \cdot \widetilde{OPT} \geq OPT$$

Multiple items – Positive result

- **Many buyers:**
  - **New Simultaneous item auction (Simultaneous privatized second price auction – SP2PA)**
    - Privatized values $\hat{v}_{il} = v_{il}(s_i, 0_{-i})$
    - Allocation – each item to the highest privatized value.
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Main Result for Multiple items, many buyers regime:
under limited knowledge asymmetry, γ-heterogeneity and standard assumptions, $PoA(SP2PA) = O(d \cdot \gamma^2)$

- Same standard assumptions as before.

Multiple items – Positive result

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- Same standard **(general)** assumptions as before.
- This result also extends to the **c-SC** assumption

Multiple items – Positive result

Our Technique

Multiple items – Positive result

Our Technique

- We divide the truncated welfare into two terms:

$$\widetilde{OPT} = \underbrace{\sum_i v_i(s_i, \mathbf{0}_{-i})}_{\boldsymbol{SELF}} + \underbrace{\sum_i (\tilde{v}_i(\boldsymbol{s}) - v_i(s_i, \mathbf{0}_{-i}))}_{\boldsymbol{OTHER}}$$

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- Use the variation of the **smoothness framework**, finding different appropriate deviation for each term

Reminder:

γ-heterogeneity: $v_j(s_i + \delta, s_{-i}) - v_j(s_i, s_{-i}) \geq \boldsymbol{\gamma}(v_{j'}(s_i + \delta, s_{-i}) - v_{j'}(s_i, s_{-i}))$

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- Use the d-limited knowledge asymmetry assumption to bound the optimal welfare by the truncated welfare

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Limited knowledge asymmetry: $d \cdot \widetilde{OPT} \geq OPT$

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In more details:

- The deviation used:
    1. For bounding $\boldsymbol{SELF}$ – going all in on an item which the bidder would have won according to sampled signals.
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Together we get that:

$$O\left(\max\{\gamma, c\}^2\right) \geq \widetilde{OPT} \geq \frac{OPT}{d}$$

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  - Interdependent Values Background
  - Mechanisms Background
  - Price of Anarchy Background
- **Our Results**
  - Single item
  - **Multiple items**
    - Many buyers regime
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- Conclusion

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For **every** Simultaneous item auction, M, under no-overbidding assumption, then $PoA(M) = \tilde{\Omega}(\log n)$.

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- **Intuition** – Due to information dispersity, buyers cannot tell between high and low valued items.

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- Finally, combining all the above we get the desired result.

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- We give a strong impossibility result in **Multiple items with many items** setting

Thank you